# Chiral Lagrangians and the transition amplitude for radiative muon capture

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## Abstract

The transition operator for the radiative capture of mesons $\mu^-$ by protons is constructed starting from a chiral Lagrangian of the $N\pi\rho\, a_1\, \omega$ system obtained within the approach of hidden local symmetries. The transition operator is gauge invariant and satisfies exactly the CVC and PCAC equations.

## I. INTRODUCTION

The radiative muon capture (RMC) on proton,

$$\mu^- + p \longrightarrow n + \nu_\mu + \gamma\,, \tag{1.1}$$

has recently been measured at TRIUMF [1] for the first time. The aim of the experiment was to extract the value of the weak induced pseudoscalar constant $g_P$ with the result

$$g_P(q^2 \approx 0.88\, m_\mu^2) = (9.8 \pm 0.7 \pm 0.3)\, g_A(0)\,. \tag{1.2}$$

This value of $g_P$ is about 1.5 times larger than tho one predicted by PCAC and admitting pion-pole dominance of the induced pseudoscalar part of the weak axial current. Actually, the constant $g_P$ is the only one not well known experimentally of the four constants $g_V$,$g_M$,$g_A$ and $g_P$ entering the weak nucleon current.

There are constant efforts for many years to extract $g_P$ from the ordinay muon capture by proton,

$$\mu^- + p \longrightarrow n + \nu_\mu\,, \tag{1.3}$$

with the world average value [2] charged with a 25% error. The recent precise measurement [3] of the transition rate for the reaction

$$\mu^- + {}^3He \longrightarrow {}^3H + \nu_\mu\,, \tag{1.4}$$

lead subsequently [4,5] to the extraction of $g_P$ with an accuracy of $\approx$ 19%,

$$g_P = (1.05 \pm 0.19)\, g_P^{PCAC}\,, \tag{1.5}$$

with

$$g_P^{PCAC}(q^2) = \frac{2Mm_\mu}{q^2 + m_\pi^2}\, g_A(q^2)\,. \tag{1.6}$$

For reaction (1.3), $q^2 \approx 0.88\, m_\mu^2$ and $g_P^{PCAC} \approx 6.6\, g_A$. On the other side, the value of $q^2$ in the hadron radiative part of the amplitude for RMC can reach $q^2 \approx -m_\mu^2$ at the high end of the photon spectrum, which leads to an enhancement by a factor of 3 in the amplitude due to the induced pseudoscalar part of the weak axial interaction in this kinematical region. It is this feature of the RMC process which feeds the hope that it can be effectively used for extracting of the value of $g_P$.

The transition amplitude for the elementary reaction (1.1) was derived by numbers of authors. One can use [6] Feynman graphs obtained by attaching a $\gamma$ in every possible way to the lines of electromagnetically interacting particles participating in reaction (1.1). Then the current conservation allows one to get the hadron radiative amplitude up to terms linear in photon momentum $k$. Better way to get the amplitude is using the low energy theorems (see Refs. [7]- [11] and references therein) which provide the RMC amplitude in terms of elastic weak form factors and pion photoproduction amplitudes up to terms linear in $k$ and

$q = p_\mu - p_\nu$. The gauge invariance, CVC and PCAC are respected in this approach just to this order. Such an amplitude was applied in [1] to extract the constant $g_P$ of Eq. (1.2).

The difference between the value (1.2) of $g_P$ and its PCAC prediction opens naturally among several questions also the discussion about the structure and completeness of the applied transition amplitude. Here we present a new one obtained from the Lagrangian of the $N\pi\rho\, a_1\, \omega$ system constructed within the approach of hidden local symmetries (HLS) [12]. The advantage of this amplitude is that it satisfies gauge invariance, CVC and PCAC exactly.

In the approach of HLS [13,14], a given global symmetry group $G_g$ of a system Lagrangian is extended to a larger one by a local group $H_l$ and the Higgs mechanism generates the mass terms for gauge fields of the local group in such a way that the local symmetry is conserved. For the chiral group $G_g \equiv [SU(2)_L \times SU(2)_R]_g$ and $H_l \equiv [SU(2)_L \times SU(2)_R]_l$ the gauge particles are identified [13,14] with the $\rho$- and $a_1$ mesons. An additional extension by the group $U(1)_l$ allows one to include the isoscalar $\omega$ meson as well [15]. Moreover, external gauge fields, which are related to the electroweak interactions of the Standard Model, are included by gauging the global chiral symmetry group $G_g$.

In Sect. II, we write down the HLS Lagrangian and the associated currents necessary to construct the transition amplitude for RMC. In Sect. III, we construct this amplitude and show that it is gauge invariant and that it satisfies CVC and PCAC exactly. In Sect. IV, we compare our amplitude with the one obtained from the low energy theorems and we make our conclusion.

## II. LAGRANGIAN AND THE CURRENTS

We use the Lagrangian written in terms of the heavy meson fields representing the nonlinear realization of the HLS for the groups $H_l \equiv [SU(2)_L \times SU(2)_R]_l \times U(1)_l$ [12,15]. It can be written as

$$\begin{aligned}
\mathcal{L}_{N\pi\rho\, a_1\, \omega} = & -\bar{N}\gamma_\mu\partial_\mu N - M\bar{N}N + ig\bar{N}\gamma_5(\vec{\Pi}\,\cdot\,\vec{\tau})N - ig_\rho\frac{g_A}{2f_\pi}\bar{N}\gamma_\mu\gamma_5(\vec{\tau}\,\cdot\,\vec{\rho}_\mu\times\vec{\Pi})N \\
& -i\frac{g_\rho g_A^2}{f_\pi}\bar{N}\gamma_\mu(\vec{\tau}\,\cdot\,\vec{a}_\mu\times\vec{\Pi})N - ig_A g_\rho\bar{N}\gamma_\mu\gamma_5(\vec{\tau}\,\cdot\,\vec{a}_\mu)N \\
& -i\frac{g_\rho}{2}\bar{N}(\gamma_\mu\vec{\rho}_\mu - i\frac{\kappa_V}{4M}\sigma_{\mu\nu}\vec{\mathcal{F}}^{(\rho)}_{\mu\nu})\,\cdot\,\vec{\tau}N - i\frac{g_\omega}{2}\bar{N}(\gamma_\mu\omega_\mu - i\frac{\kappa_S}{4M}\sigma_{\mu\nu}\omega_{\mu\nu})N \\
& +i\frac{g_\rho g_A}{4f_\pi}\frac{\kappa_V}{2M}\bar{N}\gamma_5\sigma_{\mu\nu}(\vec{\Pi}\cdot\vec{\mathcal{F}}^{(\rho)}_{\mu\nu})N + i\frac{g_\omega g_A}{4f_\pi}\frac{\kappa_S}{2M}\bar{N}\gamma_5\sigma_{\mu\nu}(\vec{\Pi}\cdot\vec{\tau})\omega_{\mu\nu}N \\
& +g_\rho\vec{\rho}_\mu\cdot\vec{\Pi}\times\partial_\mu\vec{\Pi} - g_\rho\partial_\nu\vec{\rho}_\mu\cdot\vec{\rho}_\mu\times\vec{\rho}_\nu + g_\rho(\vec{\rho}_\mu\times\vec{a}_\nu - \vec{\rho}_\nu\times\vec{a}_\mu)\cdot\partial_\mu\vec{a}_\nu \\
& +\frac{1}{f_\pi}(\vec{\rho}_{\mu\nu}\cdot\vec{a}_\mu\times\partial_\nu\vec{\Pi} + \frac{1}{2}\vec{\rho}_\mu\cdot\partial_\nu\vec{\Pi}\times\vec{a}_{\mu\nu}) \\
& +\mathcal{O}(|\vec{\Pi}|^2)\,. 
\end{aligned} \tag{2.1}$$

Here

$$\vec{\mathcal{F}}^{(\rho)}_{\mu\nu} = \vec{\rho}_{\mu\nu} - g_\rho\vec{\rho}_\mu\times\vec{\rho}_\nu\,, \qquad \vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu\,, \qquad \omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu\,. \tag{2.2}$$

The Lagrangian (2.1) describes reasonably all the relevant elementary processes ($\rho\pi\pi$, $a_1\,\gamma\pi$ etc.) at intermediate energies.

The associated currents are obtained by the Glashow–Gell-Mann method [16] and read

$$J^S_{V\,,\,\mu} = -\frac{m^2_\omega}{g_\omega}\,\omega_\mu\,, \tag{2.3}$$

$$\vec{J}_{V\,,\,\mu} = -\frac{m^2_\rho}{g_\rho}\,\vec{\rho}_\mu - 2 f_\pi g_\rho\,\vec{a}_\mu \times \vec{\Pi} + \mathcal{O}(|\vec{\Pi}|^2)\,, \tag{2.4}$$

$$\begin{aligned}\vec{J}_{A\,,\,\mu} = &-\frac{m^2_\rho}{g_\rho}\,\vec{a}_\mu + f_\pi \partial_\mu \vec{\Pi} - 2 f_\pi g_\rho\,\vec{\rho}_\mu \times \vec{\Pi} \\ &+\frac{1}{g_\rho}\,(\frac{1}{2f_\pi}\,\partial_\nu \vec{\Pi} - g_\rho \vec{a}_\nu + e\vec{\mathcal{A}}_\nu) \times \vec{\rho}_{\mu\nu} + \mathcal{O}(|\vec{\Pi}|^2)\,.\end{aligned} \tag{2.5}$$

We now have the full set of vertices and currents which are necessary to construct the transition amplitude for RMC at the tree level.

## III. TRANSITION AMPLITUDE FOR RMC

The needed transition operator $T^1 - iT^2$ consists of the following terms

$$T^a(k,q) = \frac{eG}{\sqrt{2}}\,\{\,M^a(k,q)\,+\,l_\mu(0)\epsilon^\star_\nu(k)\,[\,M^{B,\,a}_{\mu\nu}(k,q)\,+\,M^a_{\mu\nu}(\pi;k,q)\,+\,M^{c.\,t.,\,a}_{\mu\nu}(a_1;k,q)\,]\,\}\,, \tag{3.1}$$

where $M^a(k,q)$ corresponds to the radiation by muon and the radiative hadron amplitude is given in the square braces as a sum of three terms. As it will become clear later, these amplitudes present actually a set of terms which satisfy separately a closed continuity equation which together provide the PCAC for the radiative hadron amplitude. In particular, $M^{B,\,a}_{\mu\nu}$ contains the nucleon Born terms (Figs. 1a and 1b) and some related contact amplitudes (Figs. 1c and 1d), $M^a_{\mu\nu}(\pi)$ contains the mesonic amplitude $M^{m.\,c.\,,a}_{\mu\nu}(k,q)$ describing the radiation of the photon by the pion in flight which was created by the weak axial current and all contact terms where an electroweak vertex (the bubble on the graph) is connected to the nucleon via pion line (Fig. 1e with the pion). Similarly, $M^{c.\,t.,\,a}_{\mu\nu}(a_1)$ is the sum of all contact terms where the electroweak interaction is connected to the nucleon by $a_1$ meson line (Fig. 1e with the $a_1$ meson).

We construct the transition amplitude for RMC using the Feynman graphs. Generally, any of amplitudes $M^a_{\mu\nu}(k,q)$ given below is related to the corresponding S-matrix element as

$$S\;=\;(2\pi)^4\,i\,\delta^4(k+q_1-q)\,l_\mu(0)\epsilon^\star_\nu(k)\,M^a_{\mu\nu}(k,q)\,. \tag{3.2}$$

The radiation by muon is

$$\begin{aligned}M^a(k,q) = &-i\bar{u}(p_\nu)\,\gamma_\mu(1+\gamma_5)S_F(p_{\mu^-}-k)i\epsilon^*_\nu(k)\gamma_\nu\,u(p_{\mu^-}) \\ &\bar{u}(p')\,\hat{J}^a_{W,\,\mu}(q_1)\,u(p)\,,\end{aligned} \tag{3.3}$$

$$\hat{J}^a_{W,\mu}(q_1) = \hat{J}^a_{V,\mu}(q_1) + \hat{J}^a_{A,\mu}(q_1)\,, \tag{3.4}$$

$$\hat{J}^a_{V,\mu}(q_1) = i m_\rho^2 \Delta^\rho_{\mu\eta}(q_1)(\gamma_\eta - \frac{\kappa_V}{2M}\sigma_{\eta\delta} q_{1\delta})\,\frac{\tau^a}{2}\,, \tag{3.5}$$

$$\hat{J}^a_{A,\mu}(q_1) = i[-g_A m_{a_1}^2\,\Delta^{a_1}_{\mu\nu}(q_1)\gamma_\nu\gamma_5 \,+\, 2iMg_A\,\Delta^\pi_F(q_1)q_{1\mu}\gamma_5]\,\frac{\tau^a}{2}\,, \tag{3.6}$$

$$\Delta^B_{\mu\nu}(l) = (\delta_{\mu\nu} + \frac{l_\mu l_\nu}{m_B^2})\,\Delta^B_F(l)\,, \qquad B = \rho,\, a_1\,, \tag{3.7}$$

$$q_1 = p' - p\,, \qquad S_F(l) = -\frac{1}{i\,l_\mu\gamma_\mu + M}\,. \tag{3.8}$$

Here $g_A = -1.26$ and our definition of the electromagnetic and weak currents conforms Ref. [17].

Let us now present the radiative hadron amplitude. The part $M^{B,\,a}_{\mu\nu}(k,q)$ is

$$M^{B,\,a}_{\mu\nu}(k,q) = \sum_{i=1}^{6}\, M^{B,\,a}_{\mu\nu}(i;k,q)\,, \tag{3.9}$$

$$\begin{aligned} M^{B,\,a}_{\mu\nu}(1) &= -\bar{u}(p')[\,\hat{J}^a_{W,\mu}(q)S_F(Q)\hat{J}^{e.\,m.}_\nu(k) \,+\, \hat{J}^{e.\,m.}_\nu(k)S_F(P)\hat{J}^a_{W,\mu}(q)\,]u(p) \\ &\equiv\, M^{B,a}_{V,\,\mu\nu}(1) \,+\, M^{B,a}_{A,\,\mu\nu}(1)\,, \end{aligned} \tag{3.10}$$

$$\begin{aligned} M^{B,\,a}_{\mu\nu}(2) &= -\frac{g_A}{2} m_\rho^2 \Delta^\rho_{\nu\zeta}(k) q_\mu \Delta^\pi_F(q)\varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p) \\ &\equiv\, -if_\pi q_\mu \Delta^\pi_F(q)\,\mathcal{M}^{B,a}_{\pi,\,\nu}(2)\,, \end{aligned} \tag{3.11}$$

$$\begin{aligned} M^{B,\,a}_{\mu\nu}(3) &= i\frac{g_A}{2}\frac{\kappa_V}{2M} m_\rho^2 \Delta^\rho_{\nu\eta}(k) q_\mu \Delta^\pi_F(q) k_\zeta \delta_{3\,a}\bar{u}(p')\gamma_5\,\sigma_{\zeta\eta}u(p) \\ &\equiv\, -if_\pi q_\mu \Delta^\pi_F(q)\,\mathcal{M}^{B,a}_{\pi,\,\nu}(3)\,, \end{aligned} \tag{3.12}$$

$$\begin{aligned} M^{B,\,a}_{\mu\nu}(4) &= -i\frac{g_A}{2}\frac{\kappa_S}{2M} m_\omega^2 \Delta^\omega_{\nu\eta}(k) q_\mu \Delta^\pi_F(q) k_\zeta \bar{u}(p')\gamma_5\,\sigma_{\zeta\eta}\tau^a u(p) \\ &\equiv\, -if_\pi q_\mu \Delta^\pi_F(q)\,\mathcal{M}^{B,a}_{\pi,\,\nu}(4)\,, \end{aligned} \tag{3.13}$$

$$M^{B,\,a}_{\mu\nu}(5) = \frac{m_\rho^4}{2}\frac{\kappa_V}{2M}\Delta^\rho_{\mu\eta}(q)\Delta^\rho_{\nu\zeta}(k)\varepsilon^{3\,a\,b}\bar{u}(p')\sigma_{\zeta\eta}\tau^b u(p)\,, \tag{3.14}$$

$$\begin{aligned} M^{B,\,a}_{\mu\nu}(6) &= -im_\rho^2\varepsilon^{3\,a\,b}\,\bar{u}(p')[\,(q_\zeta + k_\zeta)\Delta^\rho_{\mu\eta}(q)\Delta^\rho_{\nu\eta}(k)\,\hat{J}^b_{V,\zeta}(q_1) \\ &\quad - (q_{1\zeta} + q_\zeta)\Delta^\rho_{\mu\eta}(q)\Delta^\rho_{\nu\zeta}(k)\,\hat{J}^b_{V,\eta}(q_1) \,+\, (q_{1\zeta} - k_\zeta)\Delta^\rho_{\mu\zeta}(q)\Delta^\rho_{\nu\eta}(k)\,\hat{J}^b_{V,\eta}(q_1)\,]u(p)\,. \end{aligned} \tag{3.15}$$

We write also down the contribution from the radiative nucleon Born term due to the induced pseudoscalar

$$\begin{aligned} M^{B,\,a}_{ps,\,\mu\nu}(k,q) &= Mg_A q_\mu \Delta^\pi_F(q)\,\bar{u}(p')[\,\gamma_5\tau^a S_F(Q)\hat{J}^{e.\,m.}_\nu(k) \\ &\qquad + \hat{J}^{e.\,m.}_\nu(k)S_F(P)\gamma_5\tau^a\,]u(p) \,\equiv\, -if_\pi q_\mu\Delta^\pi_F(q)\,\mathcal{M}^{B,a}_{\pi,\,\nu}(1)\,, \end{aligned} \tag{3.16}$$

which serves to define the radiative pion absorption amplitude $\mathcal{M}^{B,a}_{\pi,\,\nu}(1)$. Besides presenting contact amplitudes, eqs. (3.11),(3.12) and (3.13) define also the radiative pion absorption amplitudes $\mathcal{M}^{B,a}_{\pi,\,\nu}(2)$, $\mathcal{M}^{B,a}_{\pi,\,\nu}(3)$ and $\mathcal{M}^{B,a}_{\pi,\,\nu}(4)$, respectively. In Eq. (3.10), the amplitudes $M^{B,a}_{V,\,\mu\nu}(1)$ and $M^{B,a}_{A,\,\mu\nu}(1)$ correspond to the vector-vector and axial-vector part of the weak nucleon current (3.4).

In Eqs. (3.10-3.16) the following notations are used

$$\hat{J}^{e.\,m.}_{\nu}(k) \;=\; \hat{J}^{S}_{V,\,\nu}(k) \;+\; \hat{J}^{3}_{V,\,\nu}(k)\,, \tag{3.17}$$

$$\hat{J}^{S}_{V,\,\nu}(k) \;=\; i m^2_{\omega}\Delta^{\omega}_{\nu\eta}(k)\frac{1}{2}\,(\gamma_{\eta} \;-\; \frac{\kappa_S}{2M}\,\sigma_{\lambda\eta}k_{\lambda})\,, \tag{3.18}$$

$$\hat{J}^{3}_{V,\,\nu}(k) \;=\; i m^2_{\rho}\Delta^{\rho}_{\nu\eta}(k)\frac{\tau^3}{2}\,(\gamma_{\eta} \;-\; \frac{\kappa_V}{2M}\,\sigma_{\lambda\eta}k_{\lambda})\,, \tag{3.19}$$

$$\Gamma^{b}_{\nu}(p',p) \;=\; \bar{u}(p')\,\gamma_5\gamma_{\nu}\tau^{b}\,u(p)\,. \tag{3.20}$$

We keep the amplitudes (3.10)-(3.16) together because they satisfy the following continuity equation

$$q_{\mu}\,M^{B,a}_{\mu\nu} = i f_{\pi} m^2_{\pi}\Delta^{\pi}_{F}(q)\,\sum_{i=1}^{4}\,\mathcal{M}^{B,\,a}_{\pi,\,\nu}(i) \;+\; i\varepsilon^{3\,a\,b}\bar{u}(p')\,\hat{J}^{b}_{V,\,\mu}(q_1)\,u(p)\,. \tag{3.21}$$

We further present the second part of the radiative hadron amplitude $M^{a}_{\mu\nu}(\pi;k,q)$ as

$$M^{a}_{\mu\nu}(\pi;k,q) \;=\; M^{m.\,c.\,,a}_{\mu\nu}(k,q) \;+\; \sum_{i=1}^{5}\,M^{a}_{\mu\nu}(\pi,i;k,q)\,. \tag{3.22}$$

Here

$$\begin{aligned} M^{m.\,c.\,,a}_{\mu\nu} &= -M g_A q_{\mu}\Delta^{\pi}_{F}(q) m^2_{\rho}\Delta^{\rho}_{\eta\nu}(k)\,(q_{1\eta}+q_{\eta})\Delta^{\pi}_{F}(q_1)\,\Gamma^{a}(p',p) \\ &\equiv\; -i f_{\pi} q_{\mu}\Delta^{\pi}_{F}(q)\mathcal{M}^{m.\,c.\,,a}_{\pi,\,\nu}\,, \end{aligned} \tag{3.23}$$

$$M^{a}_{\mu\nu}(\pi,1) = -2iM g_A m^2_{\rho}\Delta^{a_1}_{\mu\nu}(q)\,\Delta^{\pi}_{F}(q_1)\varepsilon^{3\,a\,b}\Gamma^{b}(p',p)\,, \tag{3.24}$$

$$M^{a}_{\mu\nu}(\pi,2) = 2iM g_A m^2_{\rho}\Delta^{a_1}_{\mu\eta}(q)\Delta^{\rho}_{F}(k)(\,k_{\eta}q_{1\nu} - q_1\cdot k\,\delta_{\eta\nu}\,)\,\Delta^{\pi}_{F}(q_1)\varepsilon^{3\,a\,b}\Gamma^{b}(p',p)\,, \tag{3.25}$$

$$M^{a}_{\mu\nu}(\pi,3) = iM g_A m^2_{\rho}\Delta^{a_1}_{F}(q)\Delta^{\rho}_{\eta\nu}(k)(\,q_{\eta}q_{1\mu} - q\cdot q_1\,\delta_{\eta\mu}\,)\,\Delta^{\pi}_{F}(q_1)\varepsilon^{3\,a\,b}\Gamma^{b}(p',p)\,, \tag{3.26}$$

$$M^{a}_{\mu\nu}(\pi,4) = 2iM g_A m^2_{\rho}\Delta^{\rho}_{\mu\nu}(k)\,\Delta^{\pi}_{F}(q_1)\varepsilon^{3\,a\,b}\Gamma^{b}(p',p)\,, \tag{3.27}$$

$$M^{a}_{\mu\nu}(\pi,5) = -iM g_A\Delta^{\rho}_{F}(k)(\,k_{\mu}q_{1\nu} - q_1\cdot k\,\delta_{\mu\nu}\,)\,\Delta^{\pi}_{F}(q_1)\varepsilon^{3\,a\,b}\Gamma^{b}(p',p)\,, \tag{3.28}$$

$$\Gamma^{b}(p',p) = \bar{u}(p')\gamma_5\tau^{b}u(p)\,. \tag{3.29}$$

The amplitude $M^{a}_{\mu\nu}(\pi;k,q)$ defined in Eq. (3.22) satisfies the continuity equation

$$q_{\mu}M^{a}_{\mu\nu}(\pi;k,q) = i f_{\pi} m^2_{\pi}\Delta^{\pi}_{F}(q)\mathcal{M}^{m.\,c.\,,a}_{\pi,\,\nu} \;-\; iM g_A q_{1\nu}\Delta^{\pi}_{F}(q_1)\varepsilon^{3\,a\,b}\Gamma^{b}(p',p)\,. \tag{3.30}$$

The radiative pion absorption amplitude $\mathcal{M}^{m.\,c.\,,a}_{\pi,\,\nu}$ is defined in Eq. (3.23) and the second term at the r. h. s. of Eq. (3.30) is simply related to the induced pseudoscalar part of the weak axial nucleon current (3.6).

The last amplitude of Eq. (3.1) we need to discuss is $M^{c.\,t.,\,a}_{\mu\nu}(a_1;k,q)$ which represents various contact terms of the $a_1$ meson range (cf. Fig. 1e with $B = a_1$). Explicitly we have

$$M^{c.\,t.,\,a}_{\mu\nu}(a_1;k,q) = \sum_{i=1}^{5}\,M^{c.\,t.,\,a}_{\mu\nu}(a_1,i;k,q)\,, \tag{3.31}$$

$$M^{c.\,t.,\,a}_{\mu\nu}(a_1,1) = \frac{1}{2}g_A q_{\mu}\Delta^{\pi}_{F}(q) m^2_{a_1}\Delta^{a_1}_{\nu\zeta}(q_1)\varepsilon^{3\,a\,b}\Gamma^{b}_{\zeta}(p',p)$$

$$\equiv -if_\pi q_\mu \Delta_F^\pi(q)\, \mathcal{M}^{c.\,t.,\,a}_{\pi,\,\nu}(a_1,1)\,, \tag{3.32}$$

$$M^{c.\,t.,\,a}_{\mu\nu}(a_1,2) = \frac{1}{2} g_A q_\mu \Delta_F^\pi(q) m^2_{a_1} \Delta^{a_1}_{\eta\zeta}(q_1) q_\lambda \,[\, k_\eta \Delta^\rho_{\lambda\nu}(k) - k_\lambda \Delta^\rho_{\eta\nu}(k)\,] \varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)$$

$$\equiv -if_\pi q_\mu \Delta_F^\pi(q)\, \mathcal{M}^{c.\,t.,\,a}_{\pi,\,\nu}(a_1,2)\,, \tag{3.33}$$

$$M^{c.\,t.,\,a}_{\mu\nu}(a_1,3) = -\frac{1}{2} g_A q_\mu \Delta_F^\pi(q) m^2_\rho \Delta^\rho_{\eta\nu}(k) q_\lambda \,[\, q_{1\eta} \Delta^{a_1}_{\lambda\zeta}(q_1) - q_{1\lambda} \Delta^{a_1}_{\eta\zeta}(q_1)\,] \varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)$$

$$\equiv -if_\pi q_\mu \Delta_F^\pi(q)\, \mathcal{M}^{c.\,t.,\,a}_{\pi,\,\nu}(a_1,3)\,, \tag{3.34}$$

$$M^{c.\,t.,\,a}_{\mu\nu}(a_1,4) = \frac{1}{2} g_A m^2_{a_1} \Delta^{a_1}_{\lambda\zeta}(q_1)[\, k_\mu \Delta^\rho_{\lambda\nu}(k) - k_\lambda \Delta^\rho_{\mu\nu}(k)\,] \varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)\,, \tag{3.35}$$

$$M^{c.\,t.,\,a}_{\mu\nu}(a_1,5) = g_A m^4_\rho \Delta^\rho_{\eta\nu}(k)\,[\,(q_\eta + q_{1\eta})\Delta^{a_1}_{\lambda\mu}(q)\Delta^{a_1}_{\lambda\zeta}(q_1) - q_\lambda \Delta^{a_1}_{\eta\mu}(q)\Delta^{a_1}_{\lambda\zeta}(q_1) + q_{1\lambda}\Delta^{a_1}_{\lambda\mu}(q)\Delta^{a_1}_{\zeta\eta}(q_1)\,]\,\varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)\,. \tag{3.36}$$

The continuity equation for the amplitude $M^{c.\,t.,\,a}_{\mu\nu}(a_1;k,q)$ of Eq. (3.31) is

$$q_\mu M^{c.\,t.,\,a}_{\mu\nu}(a_1;k,q) = if_\pi m^2_\pi \Delta^\pi_F(q) \sum_{i=1}^{3} \mathcal{M}^{c.\,t.\,,a}_{\pi,\,\nu}(a_1,i) \,+\, g_A m^2_\rho \Delta^{a_1}_{\zeta\nu}(q_1)\varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)\,. \tag{3.37}$$

The amplitudes $\mathcal{M}^{c.\,t.\,,a}_{\pi,\,\nu}(a_1, 1-3)$ are defined in Eqs. (3.32)-(3.34) and the last term at the r. h. s. of Eq. (3.37) is simply related to the contact part the weak axial nucleon current (3.6).

Summing up Eqs. (3.21),(3.30),(3.37) provides the equation of the PCAC for the radiative hadron amplitude

$$q_\mu\,[M^{B,\,a}_{\mu\nu} \,+\, M^a_{\mu\nu}(\pi) \,+\, M^{c.\,t.,\,a}_{\mu\nu}(a_1)] \,=\, if_\pi m^2_\pi \Delta^\pi_F(q)\, \mathcal{M}^a_{\pi,\,\nu} \,+\, i\varepsilon^{3\,a\,b}\bar{u}(p')\, \hat{J}^b_{W,\,\nu}(q_1)\, u(p)\,, \tag{3.38}$$

where the weak vector nucleon current $\hat{J}^b_{W,\,\mu}$ is defined in Eq. (3.4) and the full radiative pion absorption amplitude $\mathcal{M}^a_{\pi,\,\nu}$ is given by the sum of the partial radiative pion absorption amplitudes discussed in connection with Eqs. (3.21),(3.30),(3.37)

$$\mathcal{M}^a_{\pi,\,\nu} \,=\, \sum_{i=1}^{4} \mathcal{M}^{B,\,a}_{\pi,\,\nu}(i) \,+\, \mathcal{M}^{m.\,c.,\,a}_{\pi,\,\nu} \,+\, \sum_{i=1}^{3} \mathcal{M}^{c.\,t.,\,a}_{\pi,\,\nu}(a_1,i)\,. \tag{3.39}$$

Our Eq. (3.38) is in agreement with the general discussion [8] of the structure of the matrix elements of two currents.

In the next step, we verify the CVC equation for the hadron part of our RMC amplitude. For this purpose, we calculate separately the divergence of the weak vector and axial parts of this amplitude with the result

$$k_\nu\,[\,M^{B,\,a}_{V,\,\mu\nu}(1) \,+\, M^{B,\,a}_{\mu\nu}(5) \,+\, M^{B,\,a}_{\mu\nu}(6)\,] \,=\, i\varepsilon^{3\,a\,b}\bar{u}(p')\, \hat{J}^b_{V,\,\mu}(q_1)\, u(p) \tag{3.40}$$

$$k_\nu\,[\,M^{B,\,a}_{A,\,\mu\nu}(1) \,+\, \sum_{i=2}^{4} M^{B,\,a}_{\mu\nu}(i) \,+\, M^a_{\mu\nu}(\pi) \,+\, M^{c.\,t.,\,a}_{\mu\nu}(a_1)] \,=\, i\varepsilon^{3\,a\,b}\bar{u}(p')\, \hat{J}^b_{A,\,\mu}(q_1)\, u(p)\,, \tag{3.41}$$

which leads to the correct continuity equation

$$k_\nu\,[M^{B,\,a}_{\mu\nu} \,+\, M^a_{\mu\nu}(\pi) \,+\, M^{c.\,t.,\,a}_{\mu\nu}(a_1)] \,=\, i\varepsilon^{3\,a\,b}\bar{u}(p')\, \hat{J}^b_{W,\,\mu}(q_1)\, u(p)\,. \tag{3.42}$$

The gauge invariance of the combination $T^1 - iT^2$ can be now verified simply by changing $\varepsilon^*_\nu(k) \,\to\, k_\nu$ in Eqs. (3.1) and (3.3) and using Eq. (3.42).

## IV. DISCUSSION AND THE RESULTS

Equations (3.38) and (3.42) are in agreement with the general results obtained in Ref. [8] for the matrix elements of two current. Let us note that our amplitude satisfy them exactly. The derivation of such an amplitude based on the low-energy theorems [7]- [11] provides it up to the terms linear in $k$ and $q$.

As we have discussed in Ref. [12], the form of the Lagrangian (2.1) from which the RMC amplitude is constructed, is restricted at threshold by PCAC and current algebras. It follows that our amplitude coincides at threshold with the one obtained from the low-energy theorems. At higher energies (up to $1\,GeV$) it is demanded that the HLS approach incorporates vector meson dominance, respects the Weinberg sum rules and the KSFR relation and describes reasonably physical processes such as $\rho \to \pi\pi$, $a_1 \to \rho\pi$ etc. It was also shown in [12] that only at energies $\sim\, 0.8\,GeV$ one can expect sizeable effects depending on the chosen Lagrangian model. Then in the region of energies relevant for the process of RMC in nuclei one can consider our transition amplitude as reliably fixed.

Let us compare our radiative hadron amplitude with the results of Ref. [8]. We start with the amplitudes $M^{B,\,a}_{\mu\nu}(1{-}6)$ from Eqs. (3.10)-(3.15). The amplitude $M^{B,\,a}_{\mu\nu}(1)$ with the nucleon electroweak currents (3.4) and (3.17) contains the standard nucleon Born amplitude and also a contribution due to the non-pole part of the induced pseudoscalar. Actually, it comes from the transverse part of the first term of the nucleon axial current $\hat{J}^a_{A,\,\mu}$ Eq. (3.6), because the term $-ig_A\Delta^{a_1}_F(q_1)q_{1\mu}q_{1\eta}\gamma_\eta\gamma_5$ has the form of the induced pseudoscalar with the form factor $-ig_A\Delta^{a_1}_F(q_1)q_{1\eta}\gamma_\eta$. For the nucleon on the mass shell, this is effectively $2Mg_A\Delta^{a_1}_F(q_1)q_{1\mu}\gamma_5$ which allows one to write the induced pseudoscalar term with the effective form factor $\Delta^\pi_F(q_1) \to \Delta^\pi_F(q_1) - \Delta^{a_1}_F(q_1)$.

The large term $M^{B,\,a}_{\mu\nu}(2)$ of Eq. (3.11) cancels at threshold with the contact term $M^{c.\,t.,\,a}_{\mu\nu}(a_1, 1)$ of Eq. (3.32) and only higer order terms survive (see below). The amplitude $M^{B,\,a}_{\mu\nu}(3)$ does not contribute to the considered reaction. The terms $M^{B,\,a}_{\mu\nu}(4)$ and $M^{B,\,a}_{\mu\nu}(5)$ are present in Eq. (58) and Eq. (59), respectively.

The amplitude $M^{B,\,a}_{\mu\nu}(6)$ is due to vector $\rho\rho\rho$ interaction (Fig. 1d) and it contributes in the leading order to the terms linear in $k$ and $q$ as

$$\Delta\, M^{B,\,a}_{\mu\nu}(6) \,\approx\, \frac{1}{m^2_\rho}\varepsilon^{3\,a\,b}\bar{u}(p')\frac{\tau^b}{2}[\,(q_\eta + k_\eta)\gamma_\eta\,\delta_{\mu\nu} \,+\, (k_\nu - 2q_\nu)\gamma_\mu \,+\, (q_\mu - 2k_\mu)\gamma_\nu\,]u(p)\,. \quad (4.1)$$

Next we discuss the amplitudes $M^a_{\mu\nu}(\pi)$ of Eqs. (3.23)-(3.28). As we have already mentioned $M^{m.\,c.,\,a}_{\mu\nu}$ is the standard mesonic current. Having in mind that in the considered symmetry scheme $m^2_{a_1} \,=\, 2m^2_\rho$, we can sum up

$$M^a_{\mu\nu}(\pi, 1) \,+\, M^a_{\mu\nu}(\pi, 4) \,=\, iMg_A\Delta^\pi_F(q_1)\,\varepsilon^{3\,a\,b}\Gamma^b(p', p)\delta_{\mu\nu}\,. \quad (4.2)$$

Such a term is present in Eq. (59) of [8].

The amplitudes $M^a_{\mu\nu}(\pi, 2)$, $M^a_{\mu\nu}(\pi, 3)$ and $M^a_{\mu\nu}(\pi, 5)$ contribute in higher order in k and q. These terms cannot be obtained in the standard expansion technique [8].

As we have already mentioned, the large term $M^{c.\,t.,\,a}_{\mu\nu}(a_1, 1)$ compensates in the leading order $M^{B,\,a}_{\mu\nu}(2)$, the result of the sum providing the terms

$$\Delta(1) \,\equiv\, M^{B,\,a}_{\mu\nu}(2) \,+\, M^{c.\,t.,\,a}_{\mu\nu}(a_1,1) \,\approx\, \frac{g_A}{4m_\rho^2}\,q_\mu\Delta^\pi_F(q)[\,q_\nu q_\zeta - q^2\delta_{\nu\zeta}
+k_\nu k_\zeta + 2(k\cdot q)\delta_{\nu\zeta} - k_\nu q_\zeta - k_\zeta q_\nu\,]\varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)\,. \tag{4.3}$$

In the same order, the terms $M^{c.\,t.,\,a}_{\mu\nu}(a_1,2)$ and $M^{c.\,t.,\,a}_{\mu\nu}(a_1,3)$ contribute as

$$\Delta(2) \,\equiv\, M^{c.\,t.,\,a}_{\mu\nu}(a_1,2) \,+\, M^{c.\,t.,\,a}_{\mu\nu}(a_1,3) \,\approx\, \frac{g_A}{4m_\rho^2}\,q_\mu\Delta^\pi_F(q)\{\,2[\,q_\nu k_\zeta - (q\cdot k)\delta_{\nu\zeta}\,]
+(\,q^2\delta_{\nu\zeta} - q_\nu q_\zeta\,) \,+\, [\,k_\nu q_\zeta - (k\cdot q)\delta_{\nu\zeta}\,]\}\varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)\,. \tag{4.4}$$

We now sum up the results (4.3) and (4.4) and leave the terms linear in $k$ and $q$ only

$$\Delta(1) \,+\, \Delta(2) \,=\, \frac{g_A}{4m_\rho^2}\,q_\mu\Delta^\pi_F(q)[\,q_\nu k_\zeta - (q\cdot k)\delta_{\nu\zeta}\,]\varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)\,. \tag{4.5}$$

The terms of this order are present also in Eq. (58) of Ref. [8]. Our model provides the amplitudes $\bar{V}_i$ consistently.

At last, the sum of the amplitudes $M^{c.\,t.,\,a}_{\mu\nu}(a_1,4)$ and $M^{c.\,t.,\,a}_{\mu\nu}(a_1,5)$ contributes in the order

$$\Delta(3) \,=\, \frac{g_A}{4m_\rho^2}[\,(k_\mu+q_\mu)\delta_{\nu\zeta} \,-\, (2k_\zeta+q_\zeta)\delta_{\mu\nu} \,+\, (2q_\nu-k_\nu)\delta_{\mu\zeta}\,]\varepsilon^{3\,a\,b}\Gamma^b_\zeta(p',p)\,. \tag{4.6}$$

In conclusion we notice that

1. Our amplitude for RMC derived from the chiral Lagrangian of the HLS satisfies PCAC, CVC and gauge invariance exactly and coincides in the leading order with the standard one.

2. Our resulting correction terms linear in $k$ and $q$ (see Eqs. (4.1),(4.5) and (4.6)) differ from those obtained in [7]- [10] by the standard expansion technique. This is due to the different prescription for passing towards higher energies. In our approach [12], the vector meson dominance, Weinberg sum rules and KSFR relation restrict the physical amplitudes at higher energies.

3. One can obtain explicitly higher order terms from our amplitude, which is not possible using the low energy expansion technique.

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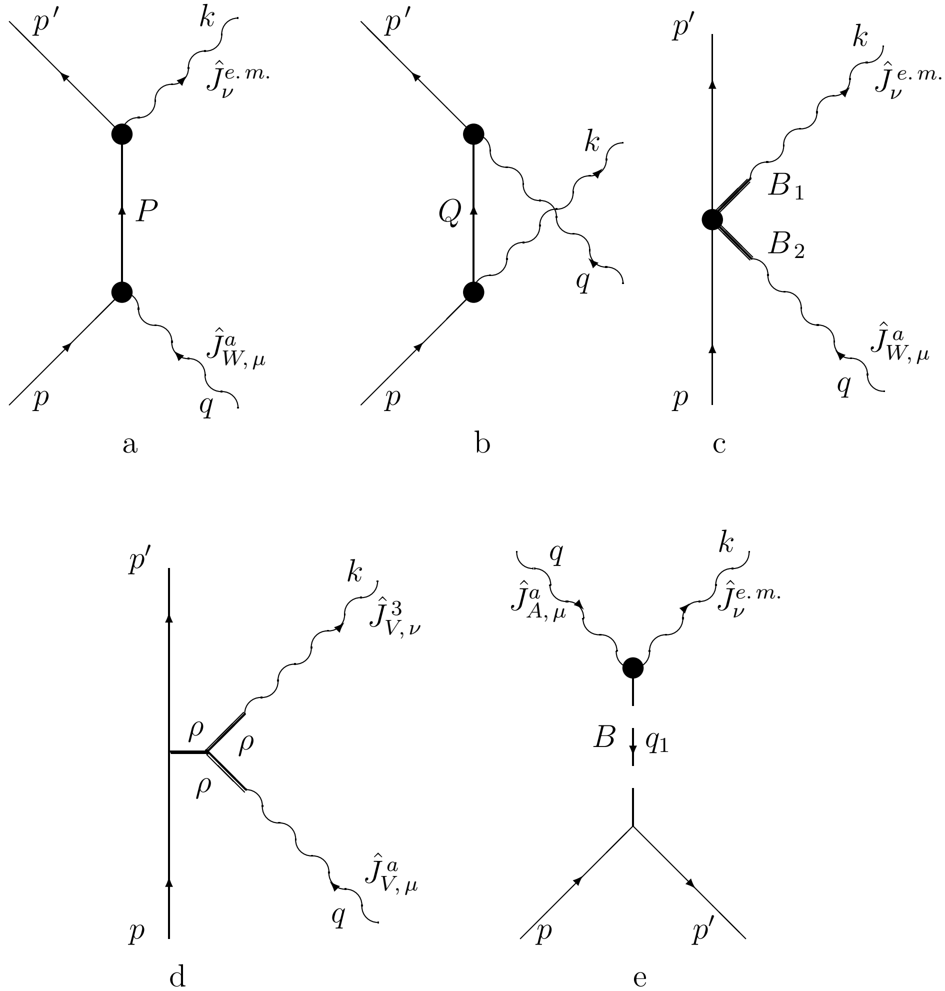


Fig. 1. The radiative hadron amplitude. Graphs a,b – the nucleon Born terms; graphs c,d – the contact terms related to the nucleon Born terms, the possible pairs $(B_1, B_2)$ are $\rho, \pi$, $\omega, \pi$ and $\rho, \rho$; graph e – the contact terms of the $B = \pi$ or $a_1$ range.